TOPIC 2
MOTION, FORCE AND ENERGY

2.1 Newton's laws of motion

2.1.1 Speed, velocity and acceleration

When something is moving, there are several questions we can ask about its motion. How fast is it travelling? Which way is it going? Is it getting faster or slower? Is it changing direction?

The answer to 'how fast …?' can be found using

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}} \tag{2.1a}$$

In symbols, with u the speed, d the distance travelled and t the time taken:

$$u = \frac{d}{t} \tag{2.1b}$$

If the **speed** is constant, then equal distances are covered in equal times and a graph of distance d against time t is a straight line. Figure 2.1 shows a graph of d against t for a car travelling at a constant speed of $20\,\text{m}\,\text{s}^{-1}$, that is in each second, its distance increases by 20 m.

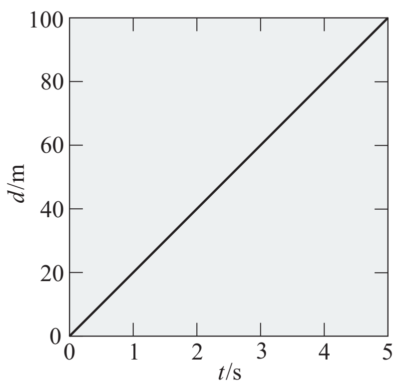

Figure 2.1 A graph of distance against time for a car travelling at a constant speed of $20\,\text{m}\,\text{s}^{-1}$.

Speed alone does not fully describe the car's motion. Figure 2.2 shows the car heading due north along a straight road. A car travelling due south at $20\,\text{m}\,\text{s}^{-1}$ has the same speed (and the same graph of d against t) but its motion is different because it is moving in a different direction. To describe the motion fully we need to state both the car's speed and its direction; these two things together specify the **velocity**. So we can say that the car has a velocity of $20\,\text{m}\,\text{s}^{-1}$ due north.

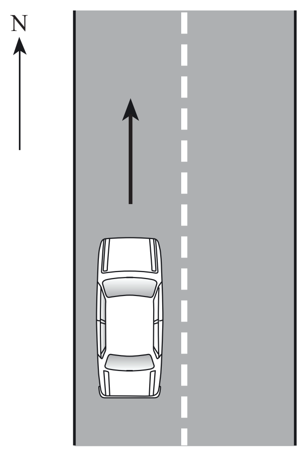

Figure 2.2 A car heading due north.

An object's **acceleration** is the rate of change of its velocity. The car in Figure 2.2 can change its velocity by changing its speed. If it speeds up to, say, $22\,\text{m}\,\text{s}^{-1}$ it is accelerating in the everyday sense of 'getting faster'. If it reduces its speed to, say, $18\,\text{m}\,\text{s}^{-1}$, then in everyday terms we would say it is decelerating (getting slower) but scientifically we use the word 'acceleration' to cover all changes of velocity regardless of whether something is speeding up or slowing down.

Now suppose the car travels round a bend at a constant speed of 20 m s^{-1}. Its direction is changing — it is no longer heading due north — and so its velocity is changing. The car is accelerating (in the scientific sense) even though it is neither speeding up nor slowing down.

QUESTION 2.1

Figure 2.3 shows the paths of four accelerating objects a–d. The dots represent the positions of the objects at equal time intervals and the arrow indicates the direction of motion. For each diagram, say whether the object is speeding up, slowing down and/or changing direction.

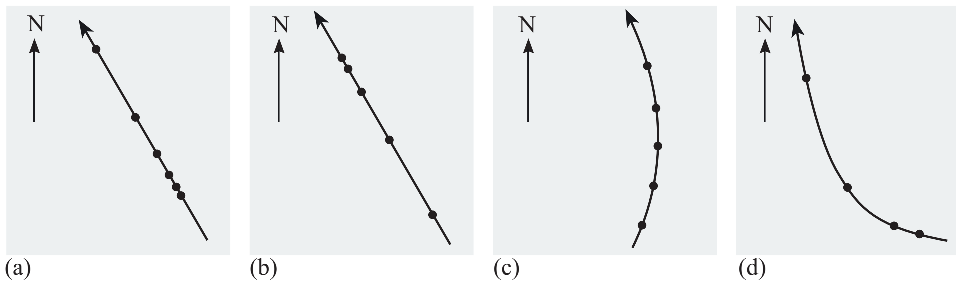

Figure 2.3 Paths of accelerating objects.

Acceleration can be calculated:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}} \tag{2.2a}$$

In symbols, with u the initial velocity and v the velocity after a time interval t, then the acceleration a is given by

$$a = \frac{v - u}{t} \tag{2.2b}$$

EXAMPLE 2.1

Suppose the car described above takes 3 seconds to accelerate from 20 m s^{-1} to 26 m s^{-1}. What is its acceleration?

Change in velocity = initial velocity – final velocity

$$= 26\ \text{m s}^{-1} - 20\ \text{m s}^{-1} = 6\ \text{m s}^{-1}$$

so

$$\text{acceleration} = \frac{6\ \text{m s}^{-1}}{3\ \text{s}} = 2\ \text{m s}^{-2}$$

Notice that the SI units of acceleration are m s^{-2}, which can be read as metres per second, per second.

If the speed decreases, then the acceleration is negative. (Calculations of acceleration when direction is changing are not required for this course. Such calculations involve maths beyond the scope of S282, S283 and this booklet.)

QUESTION 2.2

A person pushing a wheelbarrow takes 3 seconds to reach a steady walking pace of $1.5\ \mathrm{m\ s^{-1}}$, starting from rest. What is their acceleration?

2.1.2 Motion at constant velocity

If an object is at rest it will remain at rest unless anything disturbs it and makes it move, and if a moving object is undisturbed — nothing pushing or pulling on it, no friction, nothing to get in the way of its motion — it continues with constant velocity. Velocity can only change if a **force** acts on the object. The general definition of force is closely linked to acceleration: a force is that which causes acceleration. If no force acts, then velocity remains constant. Conversely, if velocity is constant, then there must be no force acting.

We have to be a careful what we mean by ‘no force’. We really mean ‘no unbalanced force’. It is possible to have two or more forces acting on a body and cancelling one another out. Force, like velocity, has both a size (a magnitude) and a direction. For two forces to cancel, they must not only have equal magnitudes but act in opposite directions. For example, if you are pushing a car on a level road the force you exert is countered by frictional forces between the car’s moving parts, as in Figure 2.4. If the frictional force exactly balances your muscular force (when the car is either at rest or in motion), the two forces cancel one another and there is no net force acting — there is no unbalanced force.

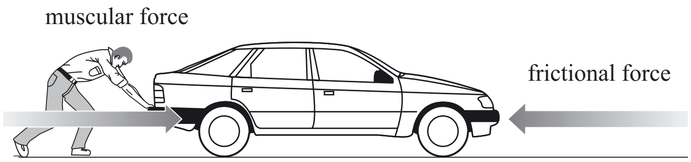

Figure 2.4 Two forces acting on a car that is being pushed.

This important result is summarized in **Newton’s first law of motion**:

> An object does not accelerate unless it is acted on by an unbalanced force.
>
> Equivalently: if an object is acted on by an unbalanced force it accelerates.

QUESTION 2.3

After some initial downward acceleration, a skydiver falls towards the Earth at constant speed. What can you deduce about any forces acting on the skydiver while she is falling vertically at constant speed?

2.1.3 Force, mass and acceleration

If, when pushing a car, your muscular force exceeds the frictional force then there is an unbalanced force and the car accelerates in the direction of that force. For a given object, the greater the unbalanced force acting on it, the greater its acceleration. But if the same force acts on objects of different mass, it produces different accelerations: the greater the mass, the smaller the acceleration produced by a given force. This enables us to define what we mean by **mass**: loosely speaking, mass is a measure of 'reluctance to accelerate'. If the mass is doubled, the acceleration produced by a given force halves, so that the product of 'mass times acceleration' stays the same.

This is an expression of **Newton's second law of motion**, and it can be stated in symbols:

$$F = ma \tag{2.3}$$

where F is the magnitude of the unbalanced force and a the acceleration it gives to a mass m.

From Newton's second law we can define a unit of force: in SI units, the unit of force (the newton, N) is the unit of mass (kg) multiplied by the unit of acceleration ($\mathrm{m\,s^{-2}}$).

EXAMPLE 2.2

A car accelerates at $1.5\,\mathrm{m\,s^{-2}}$. If the mass of the car and its occupants is 800 kg, what is the unbalanced force acting on the car?

Using $F = ma$, the unbalanced force is given by

$$F = 800\,\mathrm{kg} \times 1.5\,\mathrm{m\,s^{-2}} = 1200\,\mathrm{N} = 1.2 \times 10^3\,\mathrm{N}$$

QUESTION 2.4

Suppose that a person is sitting on a sledge on a horizontal icy surface, where the friction between the sledge and the ice is negligible. The combined mass of the person and sledge is 80 kg. What is the magnitude of the steady force that you need to apply, to accelerate the person and sledge in a straight line so that their speed increases from zero to a moderate walking pace ($1.5\,\mathrm{m\,s^{-1}}$) in 10 seconds?

QUESTION 2.5

A wind blowing on an oil-tanker of mass 2.0×10^8 kg exerts an unbalanced force of 4.0×10^6 N. What is the resulting acceleration?

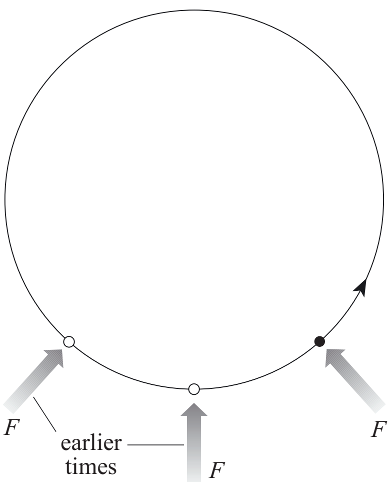

Figure 2.5 A force of constant magnitude, acting at right angles to the direction of motion, produces motion in a circle.

2.1.4 Motion in a circle

The examples (above) of pushing cars and sledges are all concerned with forces that act along, or against, the direction of motion and cause a change of speed. But what sort of force can produce an acceleration that is only a change of direction? Figure 2.5 shows how an unbalanced force can produce motion in a circle. The force is always at right angles to the direction of motion at a given instant, otherwise it would cause the speed, as well as direction, to change. Notice that the force is directed inwards, towards the centre of the circle.

The force causing circular motion can be the result of a single force, or an imbalance resulting from the inward force being greater than the outward force. Note that it is the force that causes the circular motion, not the other way around. If the force suddenly stops acting, then the object continues to move in a straight line as shown in Figure 2.6. So if you observe something moving in a circle, you know that there *must*

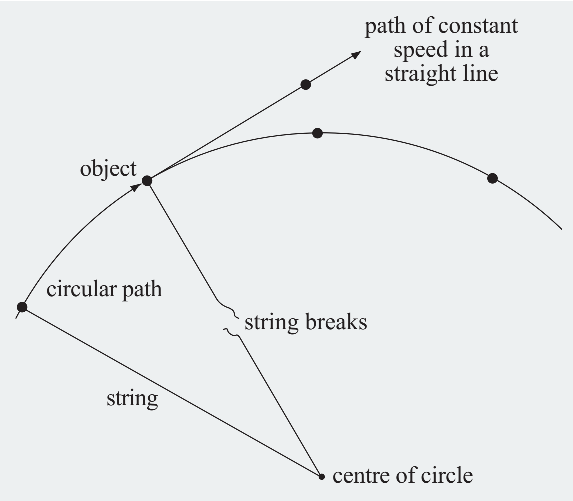

Figure 2.6 The effect on circular motion if the inward-acting force is removed, e.g. a string breaks so no inward force is applied .

be something that's producing a force on it. For example, in the case of a child whirling a ball on the end of a string, the child produces the inward force by pulling on the string.

QUESTION 2.6

What is the force responsible for keeping the Earth in its (almost) circular orbit around the Sun?

2.1.5 Forces between objects

An object can only experience a force if is interacting with some other object. *All* forces are the result of pairs of objects interacting in some way and exerting forces on one another. These pairs of forces are described by **Newton's third law of motion**.

> When two bodies interact, they exert forces on one another that are equal in magnitude and opposite in direction.

If the two interacting objects are similar in mass, then both forces can have a noticeable effect. For example, if one ice-skater pushes against another, then they will both accelerate away from one another. But if the objects are very unequal in mass, then the two resulting accelerations will be very different. For example, you are attracted towards the Earth by a gravitational force. The Earth exerts a force on you and you can experience its result: if you jump off a chair you accelerate downwards, in the direction of the force. And as described by Newton's third law, you exert a force of equal magnitude on the Earth. When you jump off a chair you accelerate towards the Earth and the Earth accelerates towards you. But the Earth's mass is so very much greater than yours that its acceleration is absolutely tiny compared with yours, and is not noticeable.

2.2 Gravitational force

2.2.1 Gravitational forces between masses

Perhaps the most important force in astronomy and planetary science is that of **gravitation**. All objects, no matter how small or large their mass, attract each other by a gravitational force. **Newton's law of gravitation**, which describes this attraction, was deduced around 1666.

Two particles, of masses m_1 and m_2 and separated by a distance r, attract each other with a gravitational force whose magnitude F_g is *proportional* to the product of their masses and *inversely proportional* to their separation. See Figure 2.7.

In symbols:

$$F_g = \frac{Gm_1m_2}{r^2} \tag{2.4}$$

where G is the gravitational constant ($G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$, notice its SI units). This law has some intuitively reasonable features. If either of the masses is increased, then F_g increases, and if the distance is increased then F_g decreases.

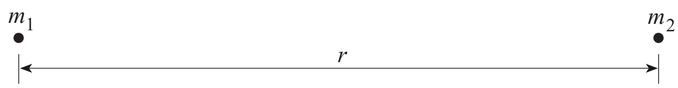

Figure 2.7 Two bodies separated by a distance r.

The separation r is the distance between the *centres* of the two objects, as shown in Figure 2.8. (Strictly, r is the distance between their centres of gravity, but for a spherical object whose mass is distributed symmetrically, such as a star or planet, the centre of gravity *is* the geometric centre.)

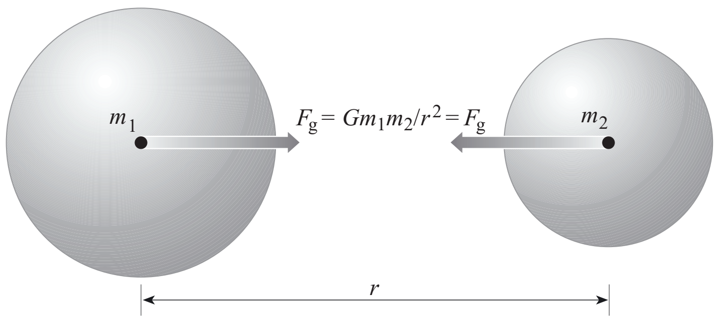

Figure 2.8 The gravitational force F_g between two spherically symmetric objects.

The gravitational force is generally noticeable only when at least one of the objects has a very large mass, as the following example and question show. But because the force is always one of attraction, gravitational forces between objects can never cancel each other out, unlike *electrical forces*, and over large distances the effects of gravity dominate over all other forces.

EXAMPLE 2.3

What is the magnitude of the gravitational force of attraction between you and the Earth when you are on the Earth's surface? The mass of the Earth is 5.97×10^{24} kg and its radius is 6.37×10^{6} m.

From Equation 2.4, if your mass is 60 kg, then

$$F_{\mathrm{g}} = \frac{6.67\times10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}\times 60\,\mathrm{kg}\times 5.97\times10^{24}\ \mathrm{kg}}{(6.37\times10^{6}\ \mathrm{m})^2} = 589\ \mathrm{N}$$

QUESTION 2.7

What is the magnitude of the gravitational force between the Earth and Moon? The Moon's mass is 7.35×10^{22} kg and the distance between the centres of the Earth and Moon is 3.85×10^{8} m.

2.2.2 Gravitational field, acceleration and weight

A mass m_1 that experiences a gravitational force due to its interaction with another mass m_2 is said to be in the gravitational field of m_2. A gravitational field can be quantified: it is defined as the gravitational force exerted on unit mass. In symbols, using g to represent the field,

$$g = \frac{F_{\mathrm{g}}}{m_1} \tag{2.5}$$

and so, from Equation 2.4

$$g = \frac{Gm_2}{r^2} \tag{2.6}$$

Like force, gravitational field has direction (towards m_2) as well as magnitude. The magnitude is often referred to as the **gravitational field strength**. From Equation 2.5 we can see that g has SI units of N kg^{-1}. The field strength thus depends on the mass m_2 and the square of the distance, r, from its centre.
The force F_{g} that an object m experiences due to a gravitational field is called its **weight**. So from Equation 2.5 we have

$$F_{\mathrm{g}} = mg \tag{2.7}$$

EXAMPLE 2.4

Using information from Example 2.3, write down your weight when you are on the Earth's surface.

No calculation is necessary. The force calculated in Example 2.3 *is* your weight. So if your mass is 60 kg then your weight is 589 N.

EXAMPLE 2.5

Using information from Example 2.4, what is the Earth's surface gravity, i.e. the strength of the gravitational field at its surface?

From Equation 2.7

$$g = \frac{F_{\mathrm{g}}}{m_1} = \frac{589\ \mathrm{N}}{60\ \mathrm{kg}} = 9.8\ \mathrm{N\,kg^{-1}}$$

(Note that the result is the same irrespective of the mass.)

When an object is in a gravitational field, and is not experiencing any forces other than its own weight, it will accelerate in the direction of the field. Its **acceleration due to gravity** (also called the acceleration of free fall) is found by rearranging Equation 2.3, which yields something remarkably similar to Equation 2.5. Using g to represent this gravitational acceleration we can see that it is really the same as field strength, the only (subtle) distinction being that the gravitational field is there all the time, whereas acceleration occurs only when something is falling freely under the influence of gravity.

From Example 2.2 and related text, we can see that the SI units of gravitational field strength ($\mathrm{N\,kg^{-1}}$) are exactly equivalent to those of acceleration ($\mathrm{m\,s^{-2}}$) because $1\,\mathrm{N} = 1\,\mathrm{kg\,m\,s^{-2}}$. So we can say both that the gravitational field at the Earth's surface has magnitude $9.8\,\mathrm{N\,kg^{-1}}$ and that the acceleration due to gravity at the Earth's surface is $9.8\,\mathrm{m\,s^{-2}}$.

QUESTION 2.8

The surface gravity of Mars is $3.7\,\mathrm{N\,kg^{-1}}$. What is the acceleration due to gravity on the surface of Mars? What would be the weight of an astronaut, mass 80 kg, standing on the Martian surface? What is the weight of the same astronaut on the Earth's surface, where $g = 9.8\,\mathrm{N\,kg^{-1}}$?

QUESTION 2.9

On the Moon, an astronaut of mass 75 kg has weight 120 N. What is the strength of the Moon's gravitational field at its surface? The astronaut drops a hammer. What is the hammer's acceleration as it falls?

2.3 Electrical force

If two objects carry an electric **charge**, they will interact by exerting a force on each other which is usually much stronger than their gravitational attraction: the **electrical force**. In SI units, electric charge is measured in coulombs (C), named after the French scientist Charles de Coulomb (1736–1806). If one object has a charge Q_1 and another has a charge Q_2, then the magnitude of the electrical force F_e between them is given by **Coulomb's law**:

$$F_e = \frac{-k_e Q_1 Q_2}{r^2} \tag{2.8}$$

where k_e is a constant that depends on the medium between the charges and r is the distance between them. The sign of Q_1Q_2 determines the direction of the force. If both charges have the same sign (both positive or both negative, so that Q_1Q_2 is positive and F_e negative) then the force is repulsive, whereas if they are of opposite sign (Q_1Q_2 negative, F_e positive) then the force is attractive.

Equation 2.8 applies to point charges, or to two spherically symmetric charge distributions with r being the distance between their centres. In a vacuum, the constant k_e is written, for historical reasons, as $1/4\pi\varepsilon_0$, where ε_0 is called the permittivity of free space. $1/4\pi\varepsilon_0$ has the value $8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$ and ε is the Greek letter epsilon.

Coulomb's law is similar in many ways to Newton's law of gravitation. It depends on the charges of the two objects and on their separation, and in an analogous way to gravitational field we can define the electric field due to any electric charge Q: it is the electrical force per unit charge.

There are, however, some big differences between the two laws. First, whereas mass can only be positive, electric charge can be positive or negative. The second big difference is that, whereas all objects exert gravitational forces on each other, many have zero electric charge — they are electrically neutral — and therefore they experience no electrical force: if either Q_1 or Q_2 in Equation 2.8 is zero, then F_e is also zero. Another big difference is that the electric force can be reduced by placing suitable material between the two charges whereas gravitational force is unaffected by any intervening material. And, not least, the constant k_e is very much greater than G, so between two charged objects reasonably close to each other, the magnitude of the electric force almost always far outweighs that of the gravitational force.

EXAMPLE 2.6

Calculate the magnitude of the electrical force between an electron (charge $Q_1 = -1.60 \times 10^{-19}$ C) and a proton (charge $Q_2 = +1.60 \times 10^{-19}$ C) in a hydrogen atom where they are separated by a distance 5.29×10^{-11} m. Is the force attractive or repulsive?

From Equation 2.8

$$
\begin{aligned}
F_e &= \frac{-k_e Q_1 Q_2}{r^2} \\
&= \frac{-(8.99\times10^9\ \text{N m}^2\ \text{C}^{-2})\times(-1.60\times10^{-19}\ \text{C}\times1.60\times10^{-19}\ \text{C})}{(5.29\times10^{-11}\ \text{m})^2} \\
&= 8.22\times10^{-8}\ \text{N}
\end{aligned}
$$

The force is attractive, because Q_1 and Q_2 have opposite sign, making F_e positive.

QUESTION 2.10

What is the magnitude and direction of the force between two protons, each carrying a charge of $+1.60 \times 10^{-19}$ C and separated by a distance 1.0×10^{-12} m? What would be the force between two helium nuclei, each with a charge twice that of a proton, separated by the same distance?

2.4 Energy

2.4.1 Energy conservation

One of the most important and widespread concepts in science and technology is that of **energy**. The scientific concept is, in essence, the same as the everyday concept — energy is a measure of the capacity of a body to make things happen.

Energy takes many forms. For example, an object has energy by virtue of its motion: a cannon-ball in flight clearly has the capacity to make things happen when it hits something. On the other hand, a cannon-ball held above the ground also has the capacity to make things happen if it is allowed to fall — it has energy by virtue

of its position. A compressed spring, too, has energy, as does a mixture of gunpowder and oxygen, and the food we ingest. Stars rely on energy stored in atomic nuclei to sustain their shining. Any form of stored energy (e.g. in a raised object, in a stretched spring, in an atomic nucleus) is known as **potential energy** as it has the *potential* to make something happen at some time in the future.

When energy is transferred it might cause motion, or maybe sound and/or light is produced as in the case of a cannon-ball hitting the floor or gunpowder igniting. Most often there is also some heating — heating, light and sound are all examples of energy in transit.

Energy is an extraordinarily useful concept for two main reasons. First, practically all processes involve the transfer of energy between different locations or between different objects, and/or its conversion into different forms. Second, all forms of energy can be quantified, and energy transfers and conversions all take place according to a strict system of natural accountancy, summarized as the law of **conservation of energy**: after all the changes have taken place, you always end up with *exactly* the same amount of energy as you started with.

2.4.2 Energy and work

Imagine pushing a supermarket trolley with a constant *force* along level ground so that it gradually picks up speed. You have transferred some energy to the trolley (energy that was previously stored in your body from the food you have eaten) by doing **work**. In the scientific sense, the amount of work you do, W, is defined as the magnitude of the force F that you exert multiplied by the distance, d, that you move in the direction of the force:

$$W = Fd \tag{2.9}$$

and the amount of energy transferred is equal to the amount of work done. Equation 2.9 defines the SI units of energy and work. One joule (J), is the energy transferred when a force of 1 N moves something through 1 m in the direction of the force, so

$$1\,\text{J} = 1\,\text{N m} = 1\,\text{kg m}^2\,\text{s}^{-2}$$

The energy that you transfer by doing work on the trolley may take various forms. If there is no resistance to your pushing force (no friction in the trolley wheels) then the energy of the trolley's motion will be equal to the work you have done. But if there is friction in the wheels, some of the energy you transfer will take other forms (you might hear the wheels squeaking, and the wheel bearings will get warm) and the trolley will not move so fast.

EXAMPLE 2.7

Suppose you push a broken-down car through 15 m with a force of 415 N. How much work do you do?

Using Equation 2.9, $W = 415\,\text{N} \times 15\,\text{m} = 6.2 \times 10^3\,\text{J}$.

QUESTION 2.11

The rocket motors of a spacecraft are used to exert a force of 2.40×10^5 N and accelerate the spacecraft through a distance of 600 m. How much energy is transferred during this operation?

2.4.3 Kinetic energy

Starting from the equivalence of work and energy, and using Equation 2.9, it is possible to derive an expression for an object's **kinetic energy**, i.e. its energy of motion. An object of mass m moving at speed v has kinetic energy E_k where

$$E_k = \frac{1}{2}mv^2 \tag{2.10}$$

EXAMPLE 2.8

A golf ball of mass 5.0×10^{-2} kg moves at $80\,\mathrm{m\,s^{-1}}$. What is its kinetic energy?

From Equation 2.10,

$$E_k = \frac{5.0\times10^{-2}\ \mathrm{kg}\times(80\,\mathrm{m\,s^{-1}})^2}{2}$$
$$= 1.6\times10^2\ \mathrm{J}.$$

EXAMPLE 2.9

A rock, mass 4.0 kg, is ejected from a volcano with initial kinetic energy 800 J. What is its initial speed?

Rearranging Equation 2.10

$$v^2 = \frac{2E_k}{m}$$

so $$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2\times800\ \mathrm{J}}{4.0\ \mathrm{kg}}} = 20\,\mathrm{m\,s^{-1}}$$

QUESTION 2.12

A certain meteoroid (small rocky body) of mass 8.0 kg travels through space at $4.0 \times 10^4\,\mathrm{m\,s^{-1}}$. What is its kinetic energy?

QUESTION 2.13

How fast would an athlete, mass 60 kg, need to sprint in order to have 3000 J of kinetic energy?

2.4.4 Gravitational energy

An the object's potential energy can be increased by lifting it up. If the object is raised though a height h, by a force equal in magnitude to its weight mg, then the work done (this time against gravity) is again mgh and the potential energy is increased by this amount.

$$W = Fd = mgh \tag{2.11}$$

The *potential energy* that an object has, by virtue of its position in a *gravitational field,* is known as gravitational potential energy or **gravitational energy** and is symbolized E_g. Since only the *change* in height is important, rather than the actual height measured from some reference ground level, we usually write

$$\Delta E_g = mg\,\Delta h \tag{2.12}$$

where ΔE_g is the change in gravitational potential energy and Δh the change in height. Notice that the **delta symbol**, Δ, means 'a change in' and is *not* a number multiplying E_g or h.

EXAMPLE 2.10

Calculate the gravitational potential energy transferred to a suitcase, mass 12 kg, when it is lifted up to a luggage rack 2.0 m above the floor, in a gravitational field 9.8 N kg^{-1}.

From Equation 2.12, $\Delta E_g = 12\,\text{kg} \times 9.8\,\text{N}\,\text{kg}^{-1} \times 2.0\,\text{m} = 240\,\text{J}$. (The answer is better written as 2.4×10^2 J or 0.24 kJ, as there are only 2 *significant figures*.)

2.4.5 Motion under gravity

If an object falls freely, influenced only by gravitational force (e.g. Newton's apple), the force of gravity does work on it — the object accelerates in the direction of the force. The object gathers speed, in other words its kinetic energy increases. The kinetic energy gained by the falling object is equal to the work done on it by the force of gravity. Put another way, the falling object gains kinetic energy while losing an equal amount of the gravitational energy that it had by virtue of its height, and the total amount of energy remains unchanged as required by the law of conservation of energy.

Provided there is no transfer in other ways such as heating, the kinetic energy gained is *exactly* equal to the potential energy lost, as required by the law of conservation of energy. The overall change in energy must be zero so we can write

$$\Delta(E_g + E_k) = 0$$

or

$$\Delta E_k = -\Delta E_g \qquad (2.13)$$

The minus sign simply means that if E_g decreases (ΔE_g is negative) then E_k increases (ΔE_k is positive) and vice versa. The following example shows how to use Equation 2.13 in a calculation.

EXAMPLE 2.11

An apple of mass 0.10 kg drops from a branch at height of 2.4 m and falls freely in a gravitational field of 10 N kg^{-1}. What is its kinetic energy just before it hits the ground? How fast will it be travelling?

Its gravitational energy decreases: $\Delta E_g = -mg\,\Delta h$ (Equation 2.12) and so from Equation 2.13

$$\Delta E_k = mg\,\Delta h = 0.10\,\text{kg} \times 10\,\text{N}\,\text{kg}^{-1} \times 2.4\,\text{m} = 2.4\,\text{J}$$

Following the same method as Example 2.9 and rearranging Equation 2.10 to make v the subject:

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 2.4\,\text{J}}{0.10\,\text{kg}}} = 6.9\,\text{m}\,\text{s}^{-1}$$

As it deals entirely with changes, not absolute amounts of energy, Equation 2.13 can include situations where the initial kinetic energy is not zero.

EXAMPLE 2.12

Suppose the apple in Example 2.11 did not drop but was thrown vertically downwards with an initial velocity of 6.0 m s^{-1}. How fast will it now be moving just before it hits the ground?

$$\text{Initial } E_\text{k} = \frac{1}{2}mv^2 = \frac{0.10\,\text{kg} \times (6\,\text{m}\,\text{s}^{-1})^2}{2} = 1.8\,\text{J}$$

From Equation 2.13 and Example 2.11, its kinetic energy still increases by an amount $\Delta E_\text{k} = mg\,\Delta h = 2.4$ J, so just before it hits the ground

$$\text{Final } E_\text{k} = 2.4\,\text{J} + 1.8\,\text{J} = 4.2\,\text{J}$$

and $$v = \sqrt{\frac{2E_\text{k}}{m}} = \sqrt{\frac{2 \times 4.2\,\text{J}}{0.10\,\text{kg}}} = 9.2\,\text{m}\,\text{s}^{-1}$$

QUESTION 2.14

Part of a space probe, mass 50 kg, is plummeting out of control towards the surface of Mars. At a height of 1.0×10^4 m above the surface is it falling vertically at 200 m s^{-1}. The gravitational field is 3.7 N kg^{-1}. What is its kinetic energy just before it hits the surface? How fast is it travelling?

2.4.6 Heat and internal energy

Many energy conversions and transfers lead to a rise in temperature. Sometimes this is deliberate, as when you burn fuel to heat food or your home, and sometimes it is a by-product of some other change. For example, if you ride a bicycle the desired outcome is to use the energy stored in your body to increase your and the bicycle's kinetic energy. At the same time, friction in the bicycle's moving parts causes them to get warm, you get warm, and the road and surrounding air also become slightly warmer. If you hammer a nail, the nail and the hammer can become noticeably warm.

All temperature rises are associated with an increase in something's **internal energy**. All substances are made up of particles (atoms and molecules) that are moving randomly, and the kinetic energy associated with this random motion is one component of an object's internal energy. If we raise the temperature, we increase the kinetic energy of this random motion. The other component of internal energy is the particles' potential energy, i.e. stored energy associated with the forces between the particles. Just as increasing the separation between an object and the Earth increases their gravitational energy, so increasing the separation of particles interacting by the electric force also increases their potential energy. The internal energy is the sum of the kinetic and potential energies of all the particles.

The internal energy of an object can be increased either by doing *work* on it or supplying **heat** or by some combination of the two. Heat is energy that flows from a higher to a lower temperature because of the temperature difference, and when heat is transferred to an object the internal energy of that object increases.

In general, increasing an object's internal energy results in a temperature rise, the only exceptions being if melting or vaporization is involved. Likewise, decreasing

the internal energy generally leads to a fall in temperature. The temperature change ΔT is related to the change in internal energy (usually denoted by Δq), the mass m of the object and its **specific heat** c (also called the specific heat capacity). The specific heat is a property of the *substance* of which the object is made, and it is the change in internal energy required to bring about a temperature change of 1°C (or 1 K, measured on the absolute temperature scale) in 1 kg of the substance. It has SI units J kg^{-1} °C^{-1} or, equivalently, J kg^{-1} K^{-1}. Now ΔT is related to Δq by the equation

$$\Delta q = mc\,\Delta T \tag{2.14}$$

EXAMPLE 2.13

The specific heat of copper is 3.8×10^2 J kg^{-1} °C^{-1}. How much energy is required to heat a copper pan, mass 0.50 kg, from 20 °C to 100 °C? How much energy is required to heat a copper pan of mass 0.25 kg through the same temperature range? How much energy is given out in each case when the pan cools from 100 °C to 20 °C?

In each case ΔT = 80 °C. For the 0.50 kg copper pan, using Equation 2.14:

$$\begin{aligned}\Delta q &= 0.50\,\text{kg} \times 3.8 \times 10^2\,\text{J kg}^{-1}\,°\text{C}^{-1} \times 80\,°\text{C}\\ &= 1.5 \times 10^4\,\text{J}.\end{aligned}$$

This same amount of energy is given out when the pan cools.

If the mass is halved (0.25 kg copper) then Δq will also be halved: $\Delta q = 7.6 \times 10^3$ J (for both heating and cooling).

EXAMPLE 2.14

The specific heat of aluminium is 9.0×10^2 J kg^{-1} °C^{-1}. Will the energy required to heat an aluminium pan through the same temperature range be greater or less than that required to heat a copper pan of the same mass?

The specific heat capacity of aluminium is greater than that of copper, so more energy will be required to heat an aluminium pan of the same mass through the same temperature range.

QUESTION 2.15

Basalt rock has a specific heat capacity of about 1.2×10^3 J kg °C^{-1} and a melting temperature of about 1070 °C. How much energy is required to bring 1.0×10^3 kg of this rock from room temperature (about 20 °C) to its melting temperature? Compare this with the energy required to bring the same mass of water-ice, specific heat 2.1×10^3 J kg^{-1} °C^{-1}, from −100 °C to its melting temperature of 0 °C.

2.4.7 Changes between solid, liquid and gas states

When a solid at its melting temperature changes to a liquid at the same temperature, the particles (atoms and molecules) must acquire enough energy to move around freely rather than being fixed in position. Likewise, when a liquid at its boiling temperature becomes a *gas* at the same temperature, the particles must acquire enough energy to increase their potential energy by becoming much more

widely separated. In either case, there must be an input of energy. Conversely, when a liquid solidifies, or a gas condenses, with no change of temperature, the particles must lose energy. The energy that is transferred to or from a substance as it changes state (or phase) without changing temperature is called **latent heat**. Latent means hidden — the energy change is 'hidden' because it is not associated with a temperature change.

The energy change Δq associated with a particular change of state depends on the substance involved and the mass Δm that changes state. In symbols:

$$\Delta q = L\,\Delta m \qquad (2.15)$$

where L is the **latent heat of vaporization** of the substance, or the **latent heat of melting** (also called the **latent heat of fusion**, as appropriate). In either case, the SI units of L are $\mathrm{J\,kg^{-1}}$.

EXAMPLE 2.15

The latent heat of melting of water-ice is $3.34 \times 10^5\ \mathrm{J\,kg^{-1}}$. How much energy is required to melt 200 g (0.200 kg) of ice at its melting temperature (0 °C)?

From Equation 2.15, $\Delta q = 0.200\ \mathrm{kg} \times 3.34 \times 10^5\ \mathrm{J\,kg^{-1}} = 6.68 \times 10^4\ \mathrm{J}$.

EXAMPLE 2.16

Liquid oxygen boils at –183 °C at normal atmospheric pressure, and its latent heat of vaporization is $2.5 \times 10^5\ \mathrm{J\,kg^{-1}}$. It can be stored in a thermos flask for short periods but heat from warmer surroundings causes it to boil. If 9.5×10^4 J is transferred to a flask of liquid oxygen, what mass of oxygen is converted from liquid to gas?

Rearranging Equation 2.15:

$$\begin{aligned}\Delta m &= \frac{\Delta q}{L} \\ &= \frac{9.5\times10^4\ \mathrm{J}}{2.5\times10^5\ \mathrm{J\,kg^{-1}}} \\ &= 0.38\ \mathrm{kg}\end{aligned}$$

QUESTION 2.16

Basalt rock's latent heat of melting is about $4.8 \times 10^5\ \mathrm{J\,kg^{-1}}$. How much energy is given out when 1.0×10^3 kg of molten *basalt* from a volcano solidifies at its melting temperature (about 1070 °C)?

QUESTION 2.17

The latent heat of vaporization of water is $2.6 \times 10^6\ \mathrm{J\,kg^{-1}}$. How much water at its boiling temperature (100 °C) could be vaporized by an energy input of 5.2×10^8 J?

2.4.8 Mass and energy

Stars rely on nuclear fusion to generate their vast outputs of energy. One of the major sources of internal heating in planetary bodies (such as the Earth) is radioactive decay. Both these processes involve nuclear reactions in which the total kinetic energy of the resulting particles is much greater than that of the reactants,

and in many cases high-energy photons are also emitted. Vast quantities of energy are produced and the total mass of the products of the reaction is slightly less than that of the reacting particles. This is consistent with the law of conservation of energy, because energy and mass are not distinct quantities but can be converted into one another. In a sense, mass is a form of energy, even though we normally measure mass in quite a different way from energy and the two quantities have different dimensions. The **mass–energy equivalence** between a change in mass Δm and the corresponding change in energy ΔE is described by an equation that arises from Einstein's theory of special relativity:

$$\Delta E = c^2 \, \Delta m \tag{2.17a}$$

where c is the speed of light in a vacuum, $3.00 \times 10^8 \,\mathrm{m\,s^{-1}}$. The energy that an object has solely by virtue of its mass, as described by Equation 2.17a, is called its **rest energy** E_0 (it has this energy even when at rest, i.e. no kinetic energy)

$$E_0 = mc^2 \tag{2.17b}$$

The huge size of the factor c^2 ($= 9.00 \times 10^{16} \,\mathrm{m^2\,s^{-2}}$) means that a tiny amount of mass is equivalent to an enormous amount of energy. In a nuclear reaction less than 1% of the reacting particles' rest energy is converted into other forms but that still produces a vast output.

Reactions between subatomic particles can sometimes lead to the complete annihilation of matter to produce high-energy photons of electromagnetic radiation. In the reverse situation, matter can be produced purely from radiation.

EXAMPLE 2.17

What is the rest energy of 1.00 kg of matter? That is, if *all* the energy in 1.00 kg of matter could be converted into other forms, how much energy would that be?

Putting $m = 1.00$ kg in Equation 2.17b,

$$E_0 = 1.00\,\mathrm{kg} \times (3.00 \times 10^8 \,\mathrm{m\,s^{-1}})^2 = 9.00 \times 10^{16}\,\mathrm{J}$$

EXAMPLE 2.18

A single electron has mass 9.11×10^{-31} kg. What is its rest energy? If an electron and its antiparticle (a positron) annihilate one another, they produce two photons of equal energy. Given that a positron has exactly the same mass as an electron, what is the energy of each of the photons produced?

Using Equation 2.17b,

$$E_0 = 9.11 \times 10^{-31}\,\mathrm{kg} \times (3.00 \times 10^8 \,\mathrm{m\,s^{-1}})^2 = 8.20 \times 10^{-14}\,\mathrm{J}$$

The total energy in the annihilation is $2 \times 8.20 \times 10^{-14}$ J so the energy of each photon is 8.20×10^{-14} J.

One consequence of this equivalence between mass and energy is that, whenever an object's energy is increased (for example, by heating it or by setting it in motion) its mass also increases. However, the large size of the factor c^2 in Equation 2.17 means that in most situations the change in mass accompanying a change in energy is not noticeable. The change of mass is, however, noticeable in nuclear reactions, or when particles are accelerated to speeds close to that of light so that their kinetic energy is comparable to their rest energy.

QUESTION 2.18

In the Sun's core, hydrogen is converted into helium and about 0.7% of the mass of hydrogen is converted into other forms of energy. How much energy is produced from 1.0 kg of hydrogen?

2.4.9 Non-SI units of energy

When dealing with energies of individual particles, the quantities can be very small, as Example 2.18 showed. The **electronvolt** is a convenient non-SI unit for expressing very small energies.

$$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J} \tag{2.18a}$$

and

$$1\ \text{J} = 1\ \text{eV}/1.60 \times 10^{-19} = 6.25 \times 10^{18}\ \text{eV} \tag{2.18b}$$

Although it is a non-SI unit, multiples of eV are written using the standard SI prefixes as the following example illustrates.

EXAMPLE 2.19

The rest energy of an electron is 8.20×10^{-14} J (from Example 2.18). Express this in eV and in MeV (1 MeV = 1×10^6 eV).

$$E_0 = (8.20 \times 10^{-14} / 1.60 \times 10^{-19})\text{eV} = 5.12 \times 10^5\ \text{eV} \approx 0.5\ \text{MeV}$$

QUESTION 2.19

A hydrogen atom at room temperature has kinetic energy 6.21×10^{-21} J. What is that in eV?

2.4.10 Power

It is often useful to know the rate at which *energy* is converted or transferred. This is known as **power**. The power, P, can be calculated by dividing the energy converted, ΔE, by the time Δt taken to do the conversion:

$$P = \frac{\Delta E}{\Delta t} \tag{2.19}$$

Power has SI units of joules per second (J s^{-1}), or watts W.

$$1\ \text{W} = 1\ \text{J s}^{-1}$$

EXAMPLE 2.20

Suppose you take 30 seconds to do 6.2×10^3 J of work (e.g. pushing a car as in Example 2.7). What is your power output?

From Equation 2.19, $P = 6.2 \times 10^3\ \text{J}/30\ \text{s} = 2.0 \times 10^2\ \text{W}$.

EXAMPLE 2.21

The Sun has a power output (i.e. luminosity) of 3.83×10^{26} W. How much energy does it emit in one year (3.16×10^{7} s)?

Rearranging Equation 2.19:

$$\Delta E = P\,\Delta t$$
$$= 3.83 \times 10^{26}\ \text{W} \times 3.16 \times 10^{7}\ \text{s} = 1.21 \times 10^{34}\ \text{J}$$

QUESTION 2.20

A particular spacecraft manoeuvre (as in Question 2.11) requires an energy conversion of 1.44×10^{8} J. If this takes 30 minutes (1800 s), what is the power of the rocket engines?

QUESTION 2.21

The international space station requires a power input of about 100 kW (1.00×10^{5} W) to run its instruments. How much energy input does it need in one day (8.64×10^{4} s)?

2.5 Answers and comments for Topic 2

QUESTION 2.1

(a) Speeding up, no change of direction. (The dots get further apart. The distance travelled in a given time interval is increasing.) (b) Slowing down, no change of direction. (The dots get closer together. The distance travelled in a given time interval is decreasing.) (c) Changing direction, constant speed. (The path curves, but the dots are all the same distance apart.) (d) Speeding up and changing direction.

QUESTION 2.2

Change in velocity = $1.5\ \text{m s}^{-1} - 0\ \text{m s}^{-1} = 1.5\ \text{m s}^{-1}$

acceleration = $1.5\ \text{m s}^{-1} / 3\ \text{s} = 0.5\ \text{m s}^{-2}$

QUESTION 2.3

There is no unbalanced force acting. (The downward force of gravity is exactly countered by an upward-acting drag force caused by motion through the air.)

QUESTION 2.4

First calculate the acceleration using change in velocity ÷ time taken:

$$a = 1.5\ \text{m s}^{-1}/10\ \text{s} = 0.15\ \text{m s}^{-2}$$

Then $F = ma = 80\ \text{kg} \times 0.15\ \text{m s}^{-2} = 12\ \text{N}$.

QUESTION 2.5

Rearranging $F = ma$ we get $a = F/m$ and so

$$a = 4.0 \times 10^6\ \text{N}/2.0 \times 10^8\ \text{kg} = 2.0 \times 10^{-2}\ \text{m s}^{-2}$$

QUESTION 2.6

The gravitational attraction between the Sun and the Earth.

QUESTION 2.7

From Equation 2.4 and using the Earth's mass from Example 2.3:

$$\begin{aligned} F_g &= 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2} \times 7.35 \times 10^{22}\ \text{kg} \times 5.97 \times 10^{24}\ \text{kg}/(3.85 \times 10^8\ \text{m})^2 \\ &= 1.97 \times 10^{20}\ \text{N} \end{aligned}$$

QUESTION 2.8

Acceleration due to gravity is $g_{\text{Mars}} = 3.7\ \text{N kg}^{-1} = 3.7\ \text{m s}^{-2}$.

From Equation 2.7 and using W to represent weight:

$W_{\text{Mars}} = 80\ \text{kg} \times 3.7\ \text{N kg}^{-1} = 296\ \text{N} = 3.0 \times 10^2\ \text{N}$ (two significant figures).

$W_{\text{Earth}} = 80\ \text{kg} \times 9.8\ \text{N kg}^{-1} = 784\ \text{N} = 7.8 \times 10^2\ \text{N}$ (two significant figures).

QUESTION 2.9

From Equations 2.5 and/or 2.7, $g = F_g/m = 120\ \text{N}/75\ \text{kg} = 1.6\ \text{N kg}^{-1}$.

The acceleration due to gravity at the Moon's surface is $1.6\ \text{m s}^{-2}$.

QUESTION 2.10

From Equation 2.8, $F_e = -k_e Q_1 Q_2/r^2$

$$\begin{aligned} F_e &= -\frac{k_e Q_1 Q_2}{r^2} \\ &= \frac{-8.99 \times 10^9\ \text{N m}^2\ \text{C}^{-2} \times (1.60 \times 10^{-19}\ \text{C})^2}{(1.0 \times 10^{-12}\ \text{m})^2} \\ &= -2.3 \times 10^{-4}\ \text{N} \end{aligned}$$

The force is repulsive, because Q_1 and Q_2 both have positive signs, making F_e negative. If the charge of each particle is doubled, then the magnitude of the force is multiplied by 4, i.e. $F_e = -9.2 \times 10^{-4}\ \text{N}$.

QUESTION 2.11

From Equation 2.9, $W = 2.40 \times 10^5\ \text{N} \times 600\ \text{m} = 1.44 \times 10^8\ \text{J}$.

QUESTION 2.12

From Equation 2.10,

$$\begin{aligned} E_k &= 8.0\ \text{kg} \times (4.0 \times 10^4\ \text{m s}^{-1})^2/2 \\ &= 6.4 \times 10^9\ \text{J} \end{aligned}$$

QUESTION 2.13

Rearranging Equation 2.10 as in Example 2.9,

$$v=\sqrt{\frac{2E_\mathrm{k}}{m}}=\sqrt{\frac{2\times 3000\,\mathrm{J}}{60\,\mathrm{kg}}}=10\,\mathrm{m\,s^{-1}}$$

QUESTION 2.14

Initial $E_\mathrm{k} = \frac{1}{2}mv^2 = 50\,\mathrm{kg} \times (200\,\mathrm{m\,s^{-1}})^2/2 = 1.0 \times 10^6\,\mathrm{J}$

From Equations 2.12 and 2.13, its kinetic energy increases by an amount

$$\Delta E_\mathrm{k} = mg\,\Delta h$$

$$= 50\,\mathrm{kg} \times 3.7\,\mathrm{N\,kg^{-1}} \times 1.0 \times 10^4\,\mathrm{m} = 1.85 \times 10^6\,\mathrm{J}$$

and so final $E_\mathrm{k} = 2.85 \times 10^6\,\mathrm{J}$.

Using Equation 2.10

$$v=\sqrt{\frac{2E_\mathrm{k}}{m}}=\sqrt{\frac{2\times 2.85\times 10^6\,\mathrm{J}}{50\,\mathrm{kg}}}=336\,\mathrm{m\,s^{-1}}$$

(which should be rounded to $3.4 \times 10^2\,\mathrm{m\,s^{-1}}$ or $0.34\,\mathrm{km\,s^{-1}}$, as there are only 2 significant figures).

QUESTION 2.15

Basalt: $\Delta T = 1050\,°\mathrm{C}$. From Equation 2.14,

$$\Delta q = 1.0 \times 10^3\,\mathrm{kg} \times 1.2 \times 10^3\,\mathrm{J\,kg^{-1}\,°C^{-1}} \times 1050\,°\mathrm{C}$$

$$= 1.3 \times 10^9\,\mathrm{J}$$

Water-ice: $\Delta T = 100\,°\mathrm{C}$,

$$\Delta q = 1.0 \times 10^3\,\mathrm{kg} \times 2.1 \times 10^3\,\mathrm{J\,kg^{-1}\,°C^{-1}} \times 100\,°\mathrm{C}$$

$$= 2.1 \times 10^8\,\mathrm{J}$$

QUESTION 2.16

From Equation 2.15,

$$\Delta q = 4.8 \times 10^5\,\mathrm{J\,kg^{-1}} \times 1.0 \times 10^3\,\mathrm{kg} = 4.8 \times 10^8\,\mathrm{J}$$

QUESTION 2.17

As in Example 2.16, rearranging Equation 2.15 to make Δm the subject:

$$\begin{aligned}\Delta m &= \frac{\Delta q}{L}\\ &= \frac{5.2\times 10^8\,\mathrm{J}}{2.6\times 10^6\,\mathrm{J\,kg^{-1}}}\\ &= 2.0\times 10^2\,\mathrm{kg}\end{aligned}$$

QUESTION 2.18

Using Equation 2.17 as in Example 2.17, $\Delta m = 0.7 \times 10^{-2}$ kg $= 7 \times 10^{-3}$ kg. In Equation 2.17, $\Delta E = 7 \times 10^{-3}\ \text{kg} \times (3.00 \times 10^{8}\ \text{m s}^{-1})^2 = 6.3 \times 10^{14}$ J.

QUESTION 2.19

From Equation 2.18b, $E_k = (6.21 \times 10^{-21} / 1.60 \times 10^{-19})\text{eV} = 3.88 \times 10^{-2}$ eV.

QUESTION 2.20

Using Equation 2.19, $P = 1.44 \times 10^{8}\ \text{J}/1800\ \text{s} = 8.00 \times 10^{4}$ W.

QUESTION 2.21

Using Equation 2.19 as in Example 2.21,

$$\Delta E = 1.00 \times 10^{5}\ \text{W} \times 8.64 \times 10^{4}\ \text{s} = 8.64 \times 10^{9}\ \text{J}$$